Geometric-Algebra LMS Adaptive Filter and its Application to Rotation Estimation

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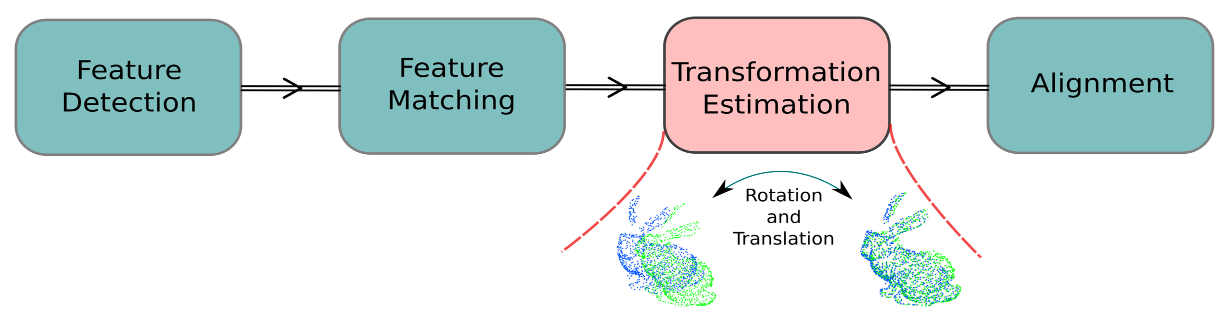
The first author was supported by CAPES Foundation, Ministry of Education of Brazil, under Grant BEX 14601/13-3. This work was conducted during Wilder Lopes' research stay at LMT, TU München.

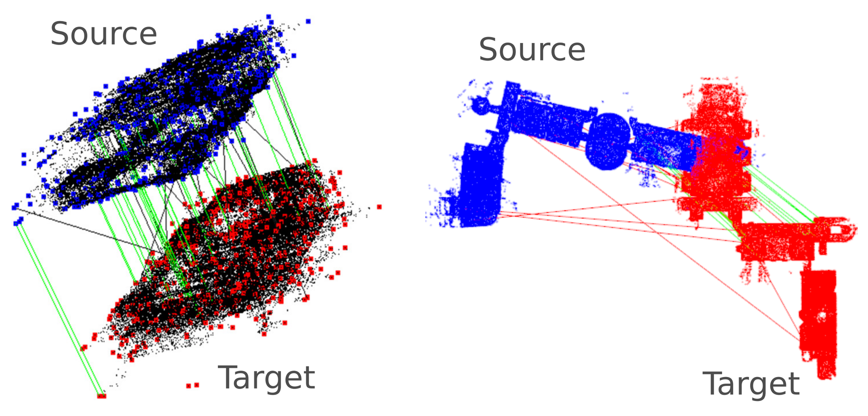
The 42nd IEEE International Conference on Acoustics, Speech and Signal Processing, March 5-9, 2017, New Orleans, USA — ICASSP2017 THE INTERNET OF SIGNALS

Introduction

- This work introduces **a new adaptive filtering technique** based on Geometric Algebra (GA) and Geometric Calculus (GC).
- **GA and GC generalize linear algebra and vector calculus** for hypercomplex variables, specially regarding the representation of geometric transformations.
- **Application in Computer Vision**: match two 3D Point Clouds (PCDs) which are initially unaligned (**3D registration**). This work focus on the "Transformation Estimation" phase, where a new estimator based on GA and adaptive filters is introduced.



3D Registration Pipeline



Pairs of Point Clouds

Given two PCDs, Y (Target) and X (Source), each with K correspondence points, what rigid transformation (rotation and translation) aligns them?

Standard Rotation Estimation

- Using linear algebra $\longrightarrow$ Least-squares problem

$x'_n \in X$ (Point in source PCD) $\quad y'_n \in Y$ (Point in target PCD)

$$\mathcal{F}(\mathbf{R}, t) = \frac{1}{K}\sum_{n=1}^{K}\left\| y'_n - \mathbf{R}x'_n - t \right\|_2^2$$

centroids: $y_n = y'_n - \bar{y} \quad x_n = x'_n - \bar{x}$

$\Rightarrow$ Minimize:

$$\mathcal{F}(\mathbf{R}) = \frac{1}{K}\sum_{n=1}^{K}\|y_n - \mathbf{R}x_n\|_2^2$$

subject to $t = \bar{y} - \mathbf{R}\bar{x}$, $\mathbf{R}\mathbf{R}^T = \mathbf{R}^T\mathbf{R} = \mathbf{I}$

Solution: SVD $\longrightarrow$ Outlier sensitive!

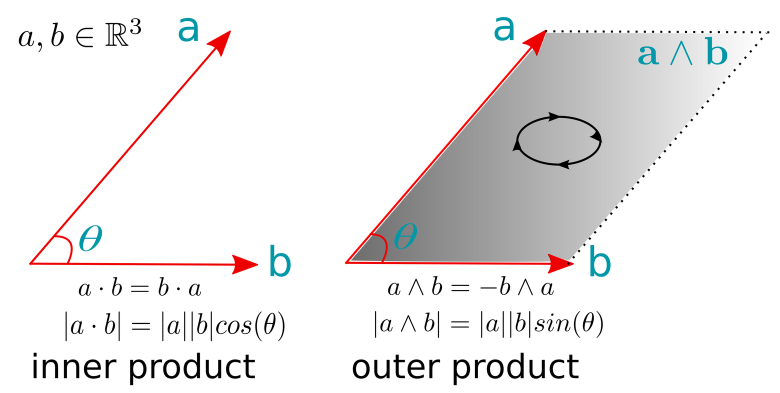
From Linear to Geometric Algebra

- Linear algebra has limitations regarding the representation of geometric structures: **inner product always results in a scalar.**
- Is it possible to construct a **new kind of product** that takes two vectors (directed lines) and returns an area (hypersurface)? Or even a vector and an area and returns a volume (hypervolume)?

Geometric Product

$$ab \triangleq a \cdot b + a \wedge b$$

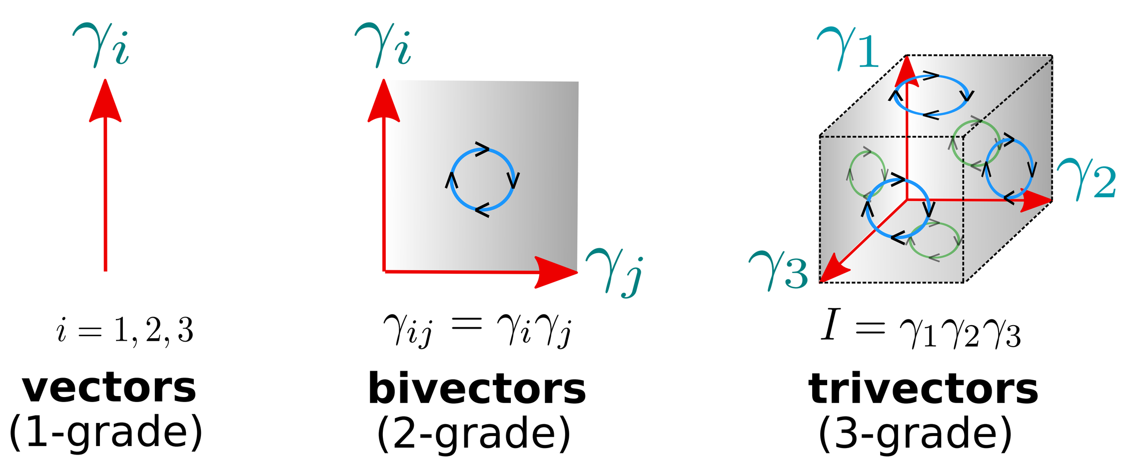
a and b in $\mathbb{R}^n$



- The entities resulting from the geometric multiplication of vectors are called **multivectors**. They are composed by their grades:

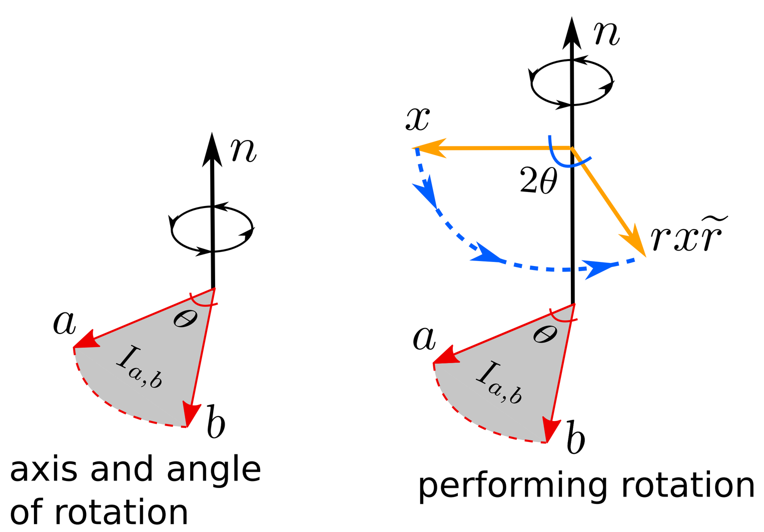
$$A = \langle A\rangle_0 + \langle A\rangle_1 + \langle A\rangle_2 + \cdots = \sum_{g=0}^{n}\langle A\rangle_g$$

scalars (0-grade), **vectors** (1-grade), **bivectors** (2-grade), g-grade; n: Dimension of vector space



- Recasting the original cost function in GA:

Performing rotation with rotors:



Rotation Matrix $\mathbf{R} \longrightarrow r(\cdot)\tilde{r}$ Rotors

New cost function:

$$J(r) = \frac{1}{K}\sum_{n=1}^{K}|y_n - rx_n\tilde{r}|^2$$

subject to

$t = \bar{y} - r\bar{x}\tilde{r} \qquad r\tilde{r} = \tilde{r}r = 1$

Reversion operation:

$\tilde{A} \triangleq \sum_{g=0}^{n}(-1)^{g(g-1)/2}\langle A\rangle_g$

GA-LMS Adaptive Filter

- Applying geometric calculus techniques:

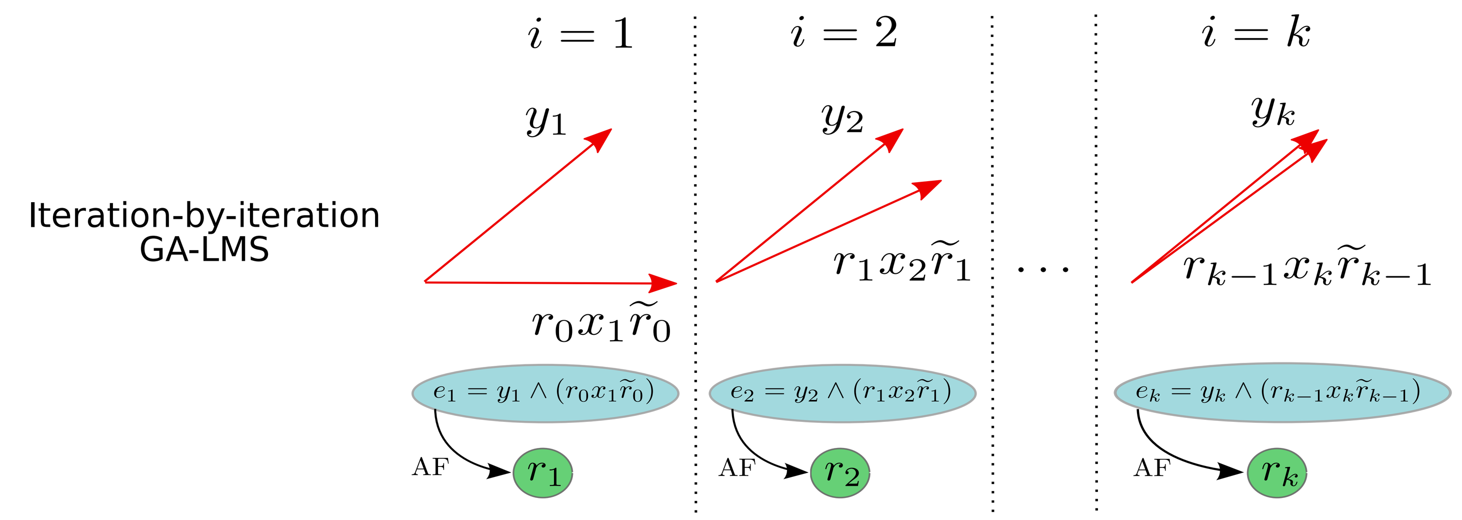
General update rule:

$$r_i = r_{i-1} + \mu G$$
$$G \triangleq -B\tilde{\nabla}J(r_{i-1})$$

Approximation by current value:

$$\nabla J(r) = 4\tilde{r}\sum_{n=1}^{K} y_n \wedge (rx_n\tilde{r}) \approx 4\tilde{r}[y_n \wedge (rx_n\tilde{r})]$$

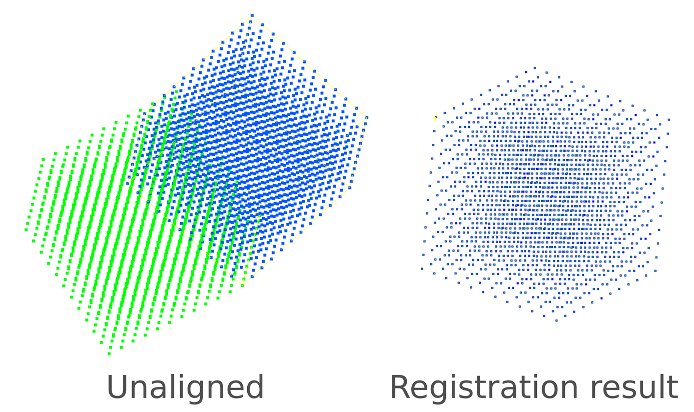
$$r_i = r_{i-1} + \mu\left[y_i \wedge (r_{i-1}x_i\tilde{r}_{i-1})\right] r_{i-1}$$

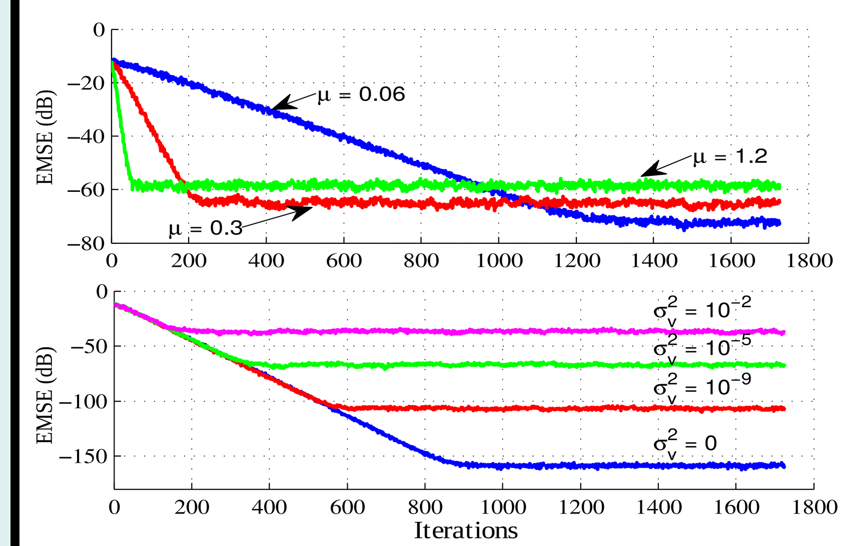


Evaluation

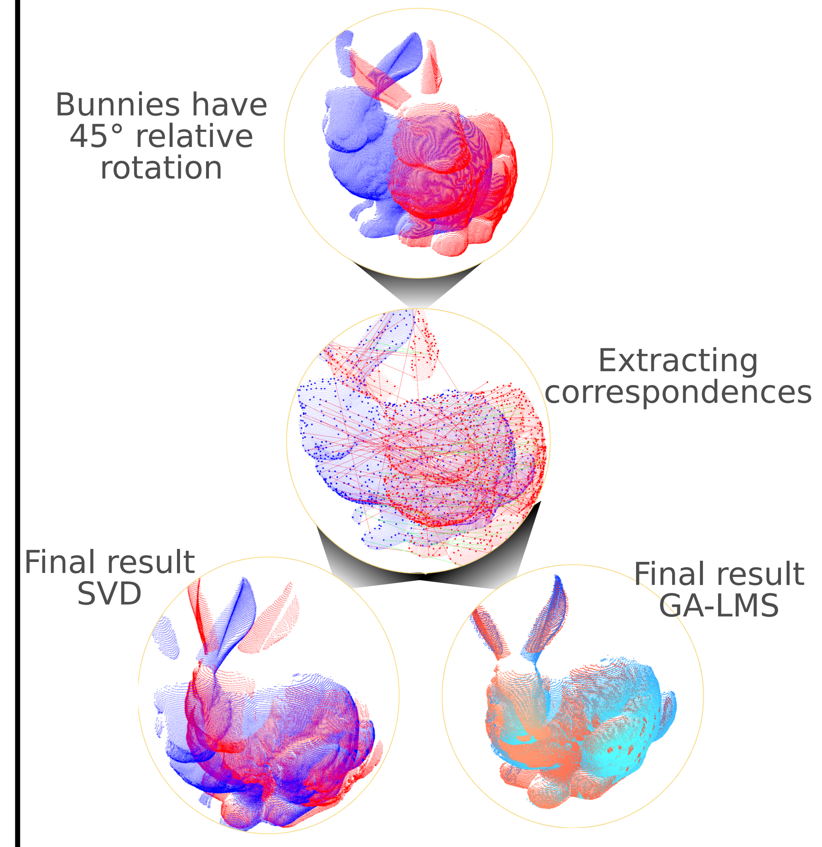
- Two scenarios:

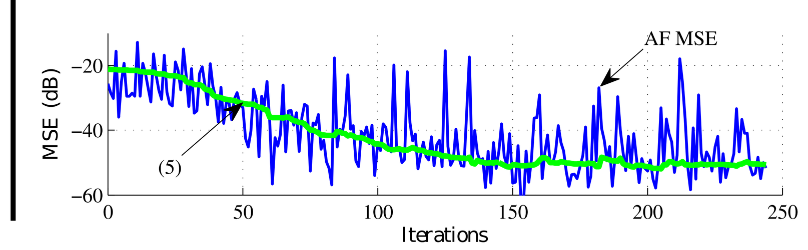
Cube registration





Bunny registration





- Codes (C++, Python, Matlab) and figures avilable on openga.org.
- See also: A. Al-Nuaimi, W. B. Lopes, E. Steinbach, C. G. Lopes, "6DOF Point Cloud Alignment using Geometric Algebra-based Adaptive Filtering", IEEE WACV 2016.